Problem Set 6: Special Relativity

3.4 (§3.1) Aberration refers to the fact that the direction of travel of a light ray depends on the motion of the observer. Hence, if a telescope observes a star at an inclination θ' to the horizontal, then show that, **classically**, the 'true' inclination θ of the star is related to θ' by

$$\tan\theta' = \frac{\sin\theta}{\cos\theta + v/c},$$

where v is the velocity of the telescope relative to the star. Show that the corresponding relativistic formula is

$$\tan\theta' = \frac{\sin\theta}{\beta(\cos\theta + v/c)}.$$

3.7 (§3.5) A particle has velocity $\boldsymbol{u} = (u_1, u_2, u_3)$ in S and $\boldsymbol{u}' = (u_1', u_2', u_3')$ in S'. Prove from the velocity transformation formulae that

$$c^2 - u^2 = \frac{c^2(c^2 - u'^2)(c^2 - v^2)}{(c^2 + u_1'v)^2}.$$

Deduce that, if the speed of a particle is less than c in any one inertial frame, then it is less than c in every inertial frame.

4.10 (§4.5) Two light rays in the xy-plane of an inertial observer, making angles θ and $-\theta$, respectively, with the positive x-axis, collide at the origin. What is the velocity v of the inertial observer (travelling in standard configuration) who sees the light rays collide head on?

4.7 (§4.4) Two particles whose rest masses are m_1 and m_2, move along a straight line with velocities u_1 and u_2, respectively, measured in the same direction. They collide inelastically to form a new particle. Show that the rest mass and velocity of the new particle are m_3 and u_3, respectively, where

$$m_3^2 = m_1^2 + m_2^2 + 2m_1 m_2 \gamma_1 \gamma_2 (1 - u_1 u_2/c^2),$$
$$u_3 = \frac{m_1 \gamma_1 u_1 + m_2 \gamma_2 u_2}{m_1 \gamma_1 + m_2 \gamma_2},$$

with

$$\gamma_1 = (1 - u_1^2/c^2)^{-1/2}, \qquad \gamma_2 = (1 - u_2^2/c^2)^{-1/2}.$$