Theory of Computation Problem Set 1

Date: 18/8/2025 Maximum Marks: 30

Problems

1. Write a rational expression and draw an automaton for the following language over the alphabet $\Sigma = \{a, b, c\}$:

$$\{w \in \Sigma^* \mid |w|_a \geq 2 \text{ iff } |w|_b \leq 2\}.$$

   Here, $|w|_a$ refers to the number of occurrences of a in w, and similarly for $|w|_b$. *(10 marks)*

2. Let Σ be a (finite) alphabet. Given a language $L \subseteq \Sigma$, define $\text{yolk}(L)$ as follows:

$$\text{yolk}(L) := \{y \mid \exists x, z \in \Sigma^* \text{ such that } |x| = |y| = |z| \text{ and } xxyzz \in L\}.$$

   Show that if L is recognizable, then $\text{yolk}(L)$ is also recognizable. *(10 marks)*

3. Let Σ be a (finite) alphabet. For words $x, y \in \Sigma^*$, we write $x \leq y$ if x is a prefix of y. For example, we have $ab \leq abcb$, but it is NOT the case that $ab \leq acb$.

   Prove that if L is a recognizable language, the set $\max(L)$ of maximal elements of L (with respect to $\leq$) is also recognizable. Prove also that if L is a recognizable language, then the set $\min(L)$ of minimal elements of L (with respect to $\leq$) is also recognizable. *(10 marks)*