Assignment 3

September 23, 2025

Due: 5 October 2025, 11:59pm Total Marks: 25

1. Let $\Sigma = \{a, b, \#\}$. Consider the language

$$L = \{(w\#)^* \mid w \in \{a, b\}^*\}.$$

That is, L consists of all strings that can be written as zero or more repetitions of a word $w \in \{a, b\}^*$ followed by the delimiter symbol '#'.

(a) is L context free? Prove. (5)

(b) Prove that the complement language $\overline{L}$ is context-free by constructing a pushdown automaton (PDA) that recognizes $\overline{L}$. (5)

2. Given a language L on the finite alphabet Σ, we define the following operators

- $cyclic(L) = \{yx \mid x, y \in \Sigma^* \ s.t. \ xy \in L\}$ (all words that are formed by cyclic-shifting letters of some word in L)
- $permutation(L) = \{y \mid \exists x \in L \ s.t. \ \forall \sigma \in \Sigma\,, \ |x|_\sigma = |y|_\sigma\}$ [1] (all words that are formed by permuting the letters of some word in L)

(a) Given a regular language R, is $cyclic(R)$: (5)

i. regular
ii. context-free, but not regular
iii. not context free

(b) Given a context-free language G, is $cyclic(G)$: (5)

i. regular
ii. context-free, but not regular
iii. not context free

(c) Given a regular language R, is $permutation(R)$: (3)

i. regular
ii. context-free, but not regular

[1] Here, $|x|_\sigma$ is the number of occurrences of σ in x

iii. not context free

(d) Given a context-free language G, is $permutation(G)$: (2)

i. regular

ii. context-free, but not regular

iii. not context free

For each part, choose the correct option out of the three and give a construction or proof as relevant.