Chennai Mathematical Institute

Design and Analysis of Algorithms CU2101

Assignment-2

Aug-Nov 2025

1. We are given n points in the unit disk, $p_i = (x_i,\ y_i)$, such that $0 < x^2 + y^2 \leq 1$ for $i \in 1, 2 \cdots, n$. Suppose that the points are uniformly distributed; that is, the probability of finding a point in any region of the circle is proportional to the area of that region. Design an algorithm with an average-case running time of $\Theta(n)$ to sort the n points by their distances $d_i = \sqrt{x_i^2 + y_i^2}$ from the origin. (Hint: Use ideas from the BUCKET-SORT.) (5)

2. Suppose that we insert n keys into a hash table of size m using open addressing and uniform hashing. Let $p(n,m)$ be the probability that no collisions occur. Show that $p(n,m) \leq e^{-n(n-1)/2m}$. Argue that when n exceeds $\sqrt{m}$, the probability of avoiding collisions goes rapidly to zero. (5)

3. Let G be a weighted undirected graph and T is a minimum spanning tree of G, suppose that we decrease the weight of one of the edges of G not in T . Give an efficient algorithm for finding the minimum spanning tree in the modified graph. (5)

4. Show that a graph has a unique minimum spanning tree if, for every cut of the graph, there is a unique *minimum weight* crossing edge. Show that the converse is not true by giving a counterexample. (5)

5. Given two positive n digit integers, a and b. Design an algorithm that multiplies a and b in $O(n^{1.58})$ time. (Hint: Use divide and conquer technique.) (5)

6. Given n chords on a circle, each defined by its endpoints. Describe an $O(n \log_2(n))$ time algorithm to determine the number of pairs of chords that intersect inside the circle. Assume that no two chords share an endpoint. For example, if the n chords are all diameters then they meet at the center and the total intersection is $\binom{n}{2}$. (5)