CALCULUS II, Fall 2025, Problem Set [1]

I

1. *Calculus on manifolds*, Spivak: Chapter 3, Exercises 1-36.
2. *Mathematical analysis*, Apostol: Exercises 14.1-14.9
3. Exercises 12-41 from the problem set of Calculus I course in Spring 2024.

II

4. *Calculus on manifolds*, Spivak: Chapter 3, Exercises 37-41.
5. *Principles of mathematical analysis*, Rudin: Chapter 10, Exercise 8.
6. Evaluate the following integrals.
   (a) $\int_A 4x + 8y$, where A is the region inside the parallelogram in the plane with vertices: (-1,3), (1, -3),(3, -1), (1,5).
   (b) $\int_A \sin(y^2)$, where A is the region in the plane bounded by the curves $x + y = 0, 2x - y = 0$ and $y = 4$.
   (c) $\int_1^2 \int_{x+2}^{x+3} \frac{1}{\sqrt{xy-x^2}}\, dy\, dx$.
   (d) $\int_0^1 \int_0^{\sqrt{x}} ye^{\sqrt{x}}\, dy\, dx$.

III

7. *Calculus on manifolds*, Spivak: Chapter 4, Exercises 1-21.
8. Let $\omega = dx_1 \wedge dx_2 + dx_3 \wedge dx_4 \in \Omega^2(\mathbb{R}^4)$. Compute $\omega \wedge \omega$.
9. Let $\omega_1, \omega_2, \omega_3 \in\in \Omega^1(\mathbb{R}^3)$ be given by $\omega_1 = x_2dx_3 - x_3dx_2$, $\omega_2 = x_3dx_1 - x_1dx_3$, $\omega_3 = x_1dx_2 - x_2dx_1$. Compute
   (a) $\omega_1 \wedge \omega_2$.
   (b) $\omega_1 \wedge \omega_3$.
   (c) $\omega_2 \wedge \omega_3$.
   (d) $\omega_1 \wedge \omega_2 \wedge \omega_3$.
10. Compute the exterior derivatives of the following differential forms.
    (a) $x_1dx_2 \wedge dx_3$.
    (b) $x_1dx_2 - x_2dx_1$.
    (c) $e^{-f}df$, where $f = x_1^2 + x_2^2 + x_3^2$.
    (d) $xdy - ydx + zdz$.
11. Solve the equation $d\eta = \omega$ for $\eta \in \Omega^1(\mathbb{R}^3)$, where ω is the 2-form given by:
    (a) $dx_2 \wedge dx_3$.

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(b) $x_2 dx_1 \wedge dx_3$.

(c) $(x_1^2 + x_2^2) dx_1 \wedge dx_2$.

(d) $\cos(x_1) dx_1 \wedge dx_3$.

12. Show that the form $\frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy \in \Omega^1(\mathbb{R}^2 \setminus \{0\})$ is closed.

13. If $\eta \in \Omega^k(\mathbb{R}^n)$ is exact and $\omega \in \Omega^l(\mathbb{R}^n)$ is closed, show that $\eta \wedge \omega$ is exact.

IV

14. *Calculus on manifolds*, Spivak: Chapter 4, Exercises 22-32.

15. Let $C \subset \mathbb{R}^2$ be a circle of radius centered at 0 and oriented counterclockwise. Show that the integral $\int_C x dy$ is equal to the area of C in two different ways: (1) using the formula for line integrals and (2) using Stokes' theorem.

16. Notes of D. Arapura:

(a) Exercise Set 1, Problem 1 (p9).

(b) Exercise Set 3, Problem 1 (p27).

(c) Exercise Set 4, Problem 1 (p35).