Quiz 1

Instructions: (1) There are no proofs of general results, only computations. But you must justify every step. (2) You may freely use the following without detailed explanation (but do show steps): definitions of all mentioned terms, basic facts about algebra of vectors and matrices, row reduction, its use in solving systems of linear equations and connection with matrix multiplication. (3) Upload your answers on moodle under Quiz 1 as instructed there. Separate pdf/jpeg file for each problem.

1. Let $M = \begin{bmatrix} \sqrt{5} & p & q \\ 0 & 0 & \pi \\ 0 & e & r \end{bmatrix}$ and $N = \begin{bmatrix} \sqrt{5} & p & q+2\pi \\ 0 & e & r \\ 0 & 0 & 3\pi \end{bmatrix}$, where e and π have their usual meaning and p, q, r are unknown real constants.

(a) If possible, find a matrix E such that $N = EM$. If such E exists, is it unique?
(b) Does there exist a matrix E' such that $N = ME'$?

2. For a 4×3 matrix M, consider the system of equations $Mx = b$ for a vector b in $\mathbb{R}^4$.

(a) Justify why there must be some b for which the system $Mx = b$ has no solution.

(b) Suppose M is reduced to a row echelon form by doing the following operations in order.

- First exchange rows 1 and 2.
- Then add row 4 to row 3, i.e. replace row 3 by (row 3 + row 4).
- Then subtract (5 times row 3) from row 2.

Explicitly find a vector b in $\mathbb{R}^4$ for which the system of equations $Mx = b$ has no solution.

3. Let $M = (m_{ij})$ with $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4, 5, 6$ be a 4×6 matrix. Its RREF R has three pivots, in columns 2, 4 and 6. Label the potentially nonzero entries of R (you should know what this means) $b_1, b_2, \ldots$, read left to right in successive rows, starting in the first row. Give the following with brief justifications. If the given information is not enough to answer a question, state so and explain.

(a) A basis of the column space of M.
(b) A basis of the row space of M.
(c) A basis of the null space of M.
(d) A basis of the null space of the transpose of M.