Analysis 1: Aug–Nov 2026

Mid-Sem Examination Makeup: Time 2 hours

Answer any 3 questions. Each question has 35 Marks.

Write answer to each question on a separate set of sheets.

Write your name and roll number of each set.

You may use definitions of *limit of a sequence, limsup and liminf of a sequence; that there exists a subsequence converging to the limsup; ratio and root tests for convergence of a series, definition of continuity and uniform continuity of a function. In problems related to continuity/uniform continuity you can use results related to sequences, after stating them precisely – not by their name. You can also use any part of any question asked in the mid-sem while answering any other part, even if you have not answered the part you are quoting.* Anything else, you have to write a proof.

(I) Let $(a_n : n \geq 1)$, $(b_n : n \geq 1)$, $(c_n : n \geq 1)$ be three sequences of real numbers such that $a_n \leq b_n \leq c_n$ for all $n \geq 1$. Suppose $(a_n : n \geq 1)$ converges to a and $(c_n : n \geq 1)$ converges to c.

[10] [a] Show that $a \leq \liminf_n b_n \leq \limsup_n b_n \leq c$.

[10] [b] If $a = c$, then show that $\lim_n b_n = a$.

[15] [c] If $a = c = 1$, writing $d_n = \frac{1}{c_n}$ if $c_n \neq 0$ and $d_n = 0$ if $c_n = 0$, show that $\lim_n d_n = 1$.

(II) Let $(a_n : n \geq 1) \subseteq \mathbb{R}$ be a sequence such that

[a] for any subsequence $(a_{n_k} : k \geq 1)$ of $(a_n : n \geq 1)$, writing $b_k = a_{n_k} : k \geq 1$, there is a subsequence $(b_{k_j} : j \geq 1)$ of $(b_k : k \geq 1)$, such that $c_j = b_{k_j}$ converges.

[b] $\exists \alpha \in \mathbb{R}$, s.t. $\forall$ subsequences $(a_{n_k} : k \geq 1)$ of $(a_n : n \geq 1)$ that converge, $(a_{n_k} : k \geq 1)$ converges to α.

[17] [i] Show that $(a_n : n \geq 1)$ is bounded.

[18] [ii] Show that $(a_n : n \geq 1)$ converges to α.

[8+8+9+10] (III) Let $f, g : [0, 1] \mapsto [0, 1]$ be continuous functions. For $x \in [0, 1]$, let $h(x) = f(x) + g(x)$, $u(x) = f(x) \times g(x)$, $v(x) = \min(f(x), g(x))$, $w(x) = f(g(x))$. Show that h, u, v, w are also continuous functions.

(IV) Let $f : \mathbb{R} \mapsto \mathbb{R}$ be a continuous function. Suppose $\lim_{x\to\infty} f(x) = u \in \mathbb{R}$ and $\lim_{x\to-\infty} f(x) = v \in \mathbb{R}$.

[17] [a] Show that f is bounded and also uniformly continuous on $\mathbb{R}$.

[18] [b] Suppose f is differentiable at x for all $x \in \mathbb{R}$, $f(0) < u$ and $f(0) < v$. Show that there exists $c \in \mathbb{R}$ such that $f'(c) = 0$.