Analysis 1: Aug–Nov 2026

Quiz 2 makeup: Time 60 minutes

Do not use any standard result(s) without providing proof.

You cannot quote a result which is essentially what you have been asked to prove.

Answer any one of the two questions. If you answer both, only first answer will be graded.

Q1: Let U, V be continuous differentiable functions defined on $\mathbb{R}$ such that:

(i) $U(0) = 0$ and $V(0) = 1$.

(ii) $U(\frac{\pi}{2}) = 1$ and $V(\frac{\pi}{2}) = 0$.

(iii) $U'(x) = V(x)$ and $V'(x) = -U(x)$.

(iv) $U^2(x) + V^2(x) = 1$.

Using Taylor's theorem, expand $U(x)$ and $V(x)$ upto n terms and show that remainder goes to 0 and thus get the power series expansion of U and V. What are the radius of convergence of these series?

Q2: Let $f : [a, b] \mapsto \mathbb{R}$ be given. Show that f is continuous if and only if for all convergent sequences $\{x_n : n \geq 1\} \subseteq [a, b]$ we have

$$\lim_{n\to\infty} f(x_n) = f(x)$$

where $x = \lim_{n\to\infty} x_n$.