Analysis 1: Aug-Nov 2026

You can use all results we have discussed on sequences and continuity of functions, as well as Rolle's theorem and Mean Value theorem- but you will have to state the result precisely. For a function h, h' will denote its derivative, if it exists.

(I) Let f, g be functions from (a, b) to $\mathbb{R}$. For $x \in (a, b)$, let f, g be differentiable at $y \in (a, b)$. Let $h(x) = f(x)g(x)$ for $x \in (a, b)$. Show that h is also differentiable at $y \in (a, b)$. Obtain an expression of h' in terms of f, g, f', g'.

(II) Let f, g be functions from (a, b) to $\mathbb{R}$. For $x \in (a, b)$, let f, g be differentiable at $y \in (a, b)$. Suppose $g(x) \neq 0$ for all $x \in (a, b)$. Let $u(x) = \frac{f(x)}{g(x)}$ for $x \in (a, b)$. Show that u is also differentiable at $y \in (a, b)$. Obtain an expression of u' in terms of f, g, f', g'.

(III) Let $f : (a, b) \mapsto (c, d)$ and $g : (c, d) \mapsto (u, v)$ be differentiable functions, with derivatives f' and g'. Let $h(x) = g(f(x))$ for $x \in (a, b)$. Show that h is also differentiable on (a, b) and its derivative h' is given by

$$h'(x) = g'(f(x))f'(x).$$

(IV) Let $E : \mathbb{R} \mapsto (0, \infty)$, $F : (-1, 1) \mapsto \mathbb{R}$ and $G : (-1, 1) \mapsto \mathbb{R}$ be defined by $E(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, $x \in \mathbb{R}$,
and for $x \in (-1, 1)$, let $F(x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k}$, $G(x) = E(F(x))$ and $H(x) = \frac{G(x)}{(1+x)}$
.

(a) Show that E, F, G, H are differentiable on their respective domains, with derivatives E', F', G' and:
$E'(x) = E(x), \ \forall x \in \mathbb{R}$,
$F'(x) = \frac{1}{1+x}, \ \forall x \in (-1, 1)$
$G'(x) = \frac{G(x)}{1+x}, \ \forall x \in (-1, 1)$
$H'(x) = 0 \ \forall x \in (-1, 1)$.

(b) For $x \in (0, 2)$, let $L(x) = F(x - 1)$ and $U(x) = E(L(x))$. Show that
$U'(x) = 1$ for $x \in (0, 2)$.
$U(x) = x$ for $x \in (0, 2)$.

(V) Let $f : (a, b) \mapsto (c, d)$ be a one-one onto (Bijective) function. Let $t : (c, d) \mapsto (a, b)$ be defined by setting $t(u) = v$ for $u \in (c, d)$ and $v \in (a, b)$ such that $f(v) = u$. Suppose f is differentiable and $f'(x) \neq 0$ for all $x \in (a, b)$. Show that t is also differentiable and obtain t' in terms of f' .